Viscosity Determination of a liquid by dropping a metal ball:

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When we drop a metal ball in a liquid, like glycerin, the motion of the ball is impeded by a resistance force from the liquid. This resistance force has two components: the buoyant force (**F_B**) and a resistive force associated with the viscosity (**F_V**) of the liquid. These two forces act in opposition to the force of gravity (**F_G**). The equation of motion will be:

$$\mathbf{F_G - F_B - F_V = M \bullet a},$$

, where the magnitude of the force of gravity is:

$$F_G = Mg = \rho_B V g,$$

, where ρ_B is the density of the ball, V is the volume of the ball and g is the magnitude of the gravity acceleration.
The magnitude of the buoyant force is:

$$F_B = \rho_L V g,$$

, where ρ_B is the density of the liquid.
The force associated with the viscosity (**F_V**) will be different depending of the speed of the ball and it takes a particularly simple expression when the speed of the ball is small. To specify what we will call a small speed, we will consider that the Reynolds number should be much smaller than 1 so that we can consider a laminar flow.

In the particular case that the liquid is glycerin at a temperature of about 20°C, the density is about 1260 Kg/m^3 , and the viscosity is about 1.49 Pa.s, considering metal balls with diameter about 4.4 mm, the Reynolds number can be calculated using the expression:

$$Re = \rho_L v d / \mu_L$$

,so if we consider that Re<1, then

$$v < \mu_L / (\rho_L * d) \approx 1.49/(1260*0.0044) = 0.268 \text{ m/s}$$

As a matter of fact, the speed of a metal ball falling in glycerin is much smaller than 0.268 m/s.

The magnitude of the resistive force associated with the viscosity (F_V) acting on a small spherical object falling at low speed in a liquid is called the Stokes's Law after George Stokes who obtained it in 1845 is given by the expression:

$$F_V = 3\pi\eta_L d v$$

, where η_L is the dynamic viscosity of the liquid, d is the diameter of the ball, and v is the velocity.

The explicit equation of motion of the ball can be written as :

$$Ma= \rho_B Va = \pi\eta_L d- \rho_L Vg - 3\pi\eta_L dv$$

We can re-write this equation as :

$$dv = (\alpha- \beta v)dt$$

where $\alpha= (\rho_B - \rho_L)g/\rho_B$ and $\beta = 18\ \eta_L/(\rho_B d^2)$.

Solving this differential equation, we get the expression for the velocity as a function of time:

$$v = (\alpha/\beta)(1-e^{-\beta t})$$

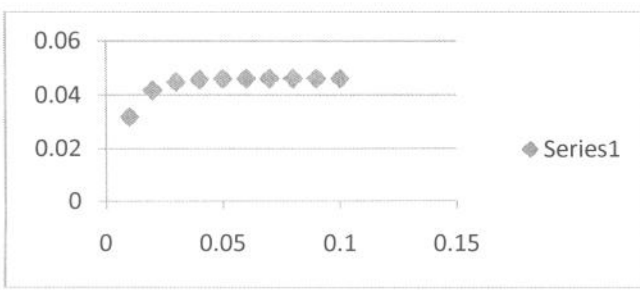
In our particular case α and β take the values:

$$\alpha = (7800-1260)*9.81/7800= 8.2\ m/s^2.$$

$$\beta = 18*(1.49)/(7800*(0.0044)^2) = 180\ s^{-1}.$$

As we can see from these results, after a very short time the ball will get the maximum velocity that will be equal to (α/β)= 0.046 m/s

In the fig you can see how the speed changes with the time. Because of the big value of β, the speed approaches to the maximum value in less than 50 msec.



EXPERIMENT:

In our experiment, we dropped small metal balls in glycerin. The temperature of the glycerin was constantly checked using a standard thermometer. We performed our experiment with a temperature of 22°C. The balls had a diameter 0.0044 m and mass 0.3486 grams, so the density of ball was calculated as 7800 Kg/m^3. After dropped, the balls traveled 40 cm inside the glycerin, distance big enough to ensure that the maximum speed was obtained, and then the time it took for the balls to travel 10 cm was measured using two photoelectric cells. The time measured for ten balls is shown in the table:

Ball	1	2	3	4	5	6	7	8	9	10	Average
Time(s)	1.6904	1.8139	1.7794	1.7784	1.7467	1.6831	1.7082	1.7157	1.7344	1.7178	1.7368

The velocity of the balls was determined by dividing the distance (0.10 m) by the average time (1.7368 s). The value obtained for the velocity is 0.058 m/s.

Using the expression for the velocity obtained before:

$$v = (\alpha/\beta)$$

We obtain the following expression for the viscosity:

$$\eta_L = (\rho_B - \rho_L) \cdot g \cdot d^2/(18 \cdot v)$$

In our case;

$$\eta_L = (7800-1260) \cdot 9.81 \cdot (0.0044)^2/18 \cdot 0.058 = 1.2 \text{ Pa.s.}$$

This result is consistent with values reported elsewhere where the values for the viscosity are 1.49 Pa.s for 20°C, and 0.95 Pa.s at 25°C. Using interpolation, we obtain:

$$(22.0-20.0)/(25.0-20.0) = (x-1.49)/(0.95-1.49),$$

$$\eta_L = 1.27 \text{ for } 22^{\circ}\text{C}$$